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APPLICATION OF AN IMPLICIT LINEAR STATISTICAL ANALYSIS TO THE ESTIMATION OF THE RESISTANCE OF A REINFORCED CONCRETE BEAM-COLUMN

by

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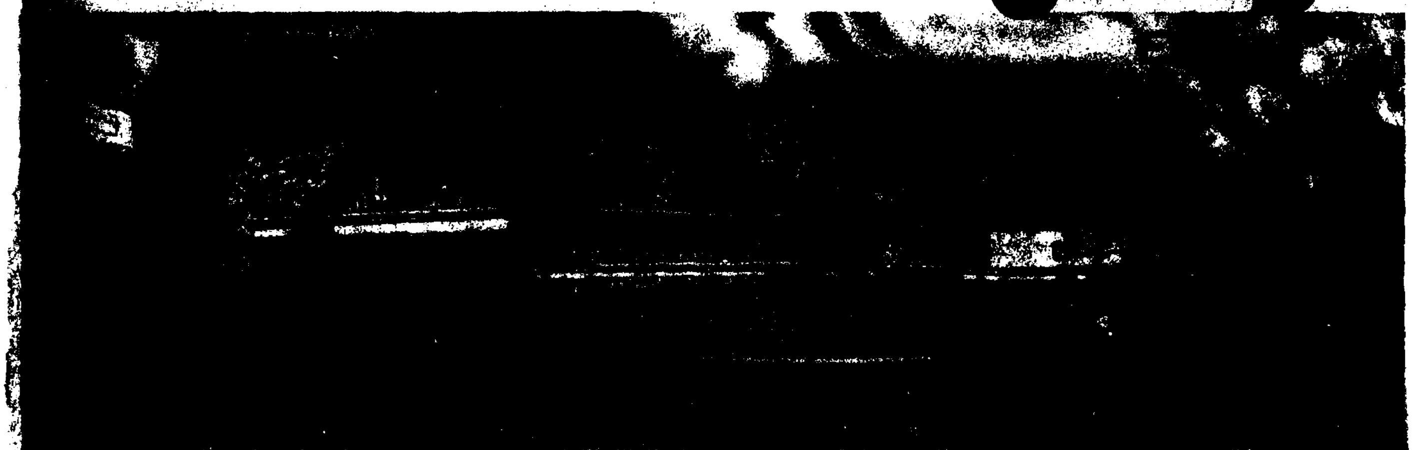

Prepared for Director
Defense Nuclear Agency
Washington, D. C. 20305

Under Subtask Y99QAXSC062 Work Unit 19

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER Miscellaneous Paper N-78-7	2. GOVT ACCESSION NO. AD-A091 373
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) APPLICATION OF AN IMPLICIT LINEAR STATISTICAL ANALYSIS TO THE ESTIMATION OF THE RESISTANCE OF A REINFORCED CONCRETE BEAM-COLUMN	5. TYPE OF REPORT & PERIOD COVERED Final report
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Paul F. Mlakar	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS U. S. Army Engineer Waterways Experiment Station Weapons Effects Laboratory P. O. Box 631, Vicksburg, Miss. 39180	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS See Block 18.
11. CONTROLLING OFFICE NAME AND ADDRESS Director Defense Nuclear Agency Washington, D. C. 20305	12. REPORT DATE November 1978
	13. NUMBER OF PAGES 30
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This work sponsored by the Defense Nuclear Agency under Subtask Y99QAXSC062, Work Unit 19, "Specific Structural Investigations."

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Concrete beams
Concrete columns
Linear statistical analysis
Reinforced concrete
Statistical analysis

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

For the moments of a function of jointly distributed random variables, an approximate solution is derived that does not require an explicit closed form expression for the derivatives of the function. The solution is based on a linear approximation of the implicit functional relation. This technique is applied to the resistance of a reinforced concrete member subjected to combined bending and thrust, and the results are compared with those obtained from Monte

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20. ABSTRACT (Continued).

Carlo simulations. The variability of this resistance due to fabrication randomness, which plays a central role in the derivation of a reliability-based design safety factor, is also discussed. Next, the larger uncertainty associated with the resistance of a beam-column, whose individual mean parameters must themselves be estimated as in the condition survey of an existing structure of unknown design, is examined. Finally, the importance of factors omitted from the resistance functional relation is considered.

PREFACE

The study presented herein was sponsored by the Defense Nuclear Agency (DNA) under Subtask Y99QAXSC062, Work Unit 19, "Specific Structural Investigations." The Technical Monitor for DNA was Dr. K. L. Goering.

The work was performed during FY 1977 at the U. S. Army Engineer Waterways Experiment Station (WES) under the general supervision of Mr. J. T. Ballard, Chief of the Structures Division (SD), and Mr. W. J. Flathau, Chief of the Weapons Effects Laboratory. This report was prepared by Dr. P. F. Mlakar, SD. The author gratefully acknowledges the many helpful discussions of this work between Mr. R. E. Walker, SD, and himself. Recognition is also given to Mr. B. K. Campbell, SD, for his valuable assistance in many of the computations.

COL J. L. Cannon, CE, was Commander and Director of the WES during the preparation of this report. Mr. F. R. Brown was Technical Director.

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UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
inches	25.4	millimetres
kips (force)	4448.222	newtons
kips (force) per square inch	6894.757	kilopascals
square inches	6.4516	square centimetres

APPLICATION OF AN IMPLICIT LINEAR STATISTICAL ANALYSIS TO THE ESTIMATION OF THE RESISTANCE OF A REINFORCED CONCRETE BEAM-COLUMN

SECTION 1

INTRODUCTION

The interaction curve for the resistance of a reinforced concrete beam-column is determined implicitly by a set of parameters, which describe the concrete strength, section geometry, reinforcement strength, and its placement.[1] In reality, these parameters are random variables, and this implies that the resistance is also a random variable. The variability of this resistance due to the basic randomness of the parameters associated with typical fabrication practices has recently been examined through a Monte Carlo simulation.[2] This examination provides information useful in the derivation of a reliability-based design safety factor, but the relative contributions of the randomness of the individual parameters warrant further study. Of additional interest in the condition survey of an existing structure of unknown design is the variability of the resistance of a beam-column whose mean values of the individual parameters must themselves be estimated. For these reasons, an implicit linear statistical analysis is developed herein and applied to the estimation of the resistance of a reinforced concrete beam-column.

SECTION 2

DETERMINISTIC ANALYSIS OF RESISTANCE

The load and moment interaction of the reinforced concrete cross section shown in Figure 2.1 has been obtained in Reference 1. Under the combined loadings of moment and thrust, it is assumed that the distribution of strain is linear as shown in the figure. Herein, the failure of the cross section will be taken to occur either if the maximum concrete strain ε_c equals the ultimate concrete strain ε_u, or if the tensile reinforcement strain ε_s equals its yield strain f_y/E_s where f_y and E_s are the yield strength and modulus of elasticity of the reinforcement, respectively. Thus, at failure

$$\varepsilon_c = \begin{cases} \dfrac{c\ f_y/E_s}{d - c} & \text{for } c < c_b \\ \varepsilon_u & \text{for } c_b \leq c \end{cases} \tag{2.1}$$

$$\varepsilon_m = \begin{cases} 0 & \text{for } c < h \\ \dfrac{c - h}{c}\,\varepsilon_c & \text{for } h \leq c \end{cases} \tag{2.2}$$

$$\varepsilon_s = (d - c)\varepsilon_c/c \tag{2.3}$$

$$\varepsilon_s' = (d' - c)\varepsilon_c/c \tag{2.4}$$

where

c = depth of the neutral axis

d, d' = depths of the tensile and compressive reinforcement, respectively

ε_m = minimum compressive strain

h = depth of the cross section

ε_s' = strain in the compressive reinforcement

and

$$c_b = \frac{\varepsilon_u d}{\varepsilon_u + f_y/E_s} \tag{2.5}$$

is the depth to the neutral axis for a balanced loading in which the concrete crushes and the tensile steel yields simultaneously. The stress-strain relation for the concrete (Figure 2.2 and Reference 3) is assumed to be

$$f_c(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon < 0 \\ \dfrac{2f''_c \; \varepsilon/\varepsilon_o}{1 + (\varepsilon/\varepsilon_o)^2} & \text{for } 0 \le \varepsilon \end{cases} \tag{2.6}$$

where f''_c is the compressive strength and ε_o is the strain corresponding to this strength. It then follows that the resultant force in the concrete is

$$N_c = f''_c \; bc \left(\frac{\varepsilon_o}{\varepsilon_c}\right) \log_e \left[\frac{1 + (\varepsilon_c/\varepsilon_o)^2}{1 + (\varepsilon_m/\varepsilon_o)^2}\right] \tag{2.7}$$

and that its moment about the neutral axis is

$$M_c = 2f''_c \; bc \left(\frac{\varepsilon_o}{\varepsilon_c}\right)^2 \left[\frac{\varepsilon_c}{\varepsilon_o} - \frac{\varepsilon_m}{\varepsilon_o} + \tan^{-1} \frac{\varepsilon_m}{\varepsilon_o} - \tan^{-1} \frac{\varepsilon_c}{\varepsilon_o}\right] \tag{2.8}$$

where b is the width of the cross section. The reinforcement stress is assumed to be linearly elasto-plastic, i.e.,

$$f_s(\varepsilon) = \begin{cases} -f_y & \text{for } \varepsilon < -f_y/E_s \\ E_s \varepsilon & \text{for } -f_y/E_s \le \varepsilon \le f_y/E_s \\ f_y & \text{for } f_y/E_s < \varepsilon \end{cases} \tag{2.9}$$

Thus, the resultant forces in the tensile and the compressive reinforcement are respectively

$$N_s = A_s \left[f_s(\varepsilon_s) - f_c(\varepsilon_s)\right] \tag{2.10}$$

$$N'_s = A'_s \left[f_x(\varepsilon'_s) - f_c(\varepsilon'_s)\right] \tag{2.11}$$

where A_s and A'_s represent the areas of the tensile and the compressive reinforcement, respectively. The axial load applied to the cross section is then simply

$$P = N_c + N_s + N'_s \tag{2.12}$$

and the resultant bending moment is

$$M = M_c + N_c(x_o - c) + N_s(x_o - d) + N'_s(x_o - d') \tag{2.13}$$

where the depth of the plastic centroid is given by

$$x_o = \frac{N_{co}(h/2) + N_{so}(d) + N'_{so}(d')}{N_{co} + N_{so} + N'_{so}} \tag{2.14}$$

in which the resultant forces in the concrete, tensile reinforcement, and compressive reinforcement for thrust without bending moment are respectively

$$N_{co} = bh\left[f_c(\varepsilon_u)\right] \tag{2.15}$$

$$N_{so} = A_s\left[f_s(\varepsilon_u) - f_c(\varepsilon_u)\right] \tag{2.16}$$

$$N'_{so} = A_s\left[f_s(\varepsilon_u) - f_c(\varepsilon_u)\right] \tag{2.17}$$

Finally, the resistance of the beam-column at fixed eccentricity

$$e = M/P \tag{2.18}$$

is defined in Reference 4 as

$$R = \sqrt{(M/h)^2 + P^2} \tag{2.19}$$

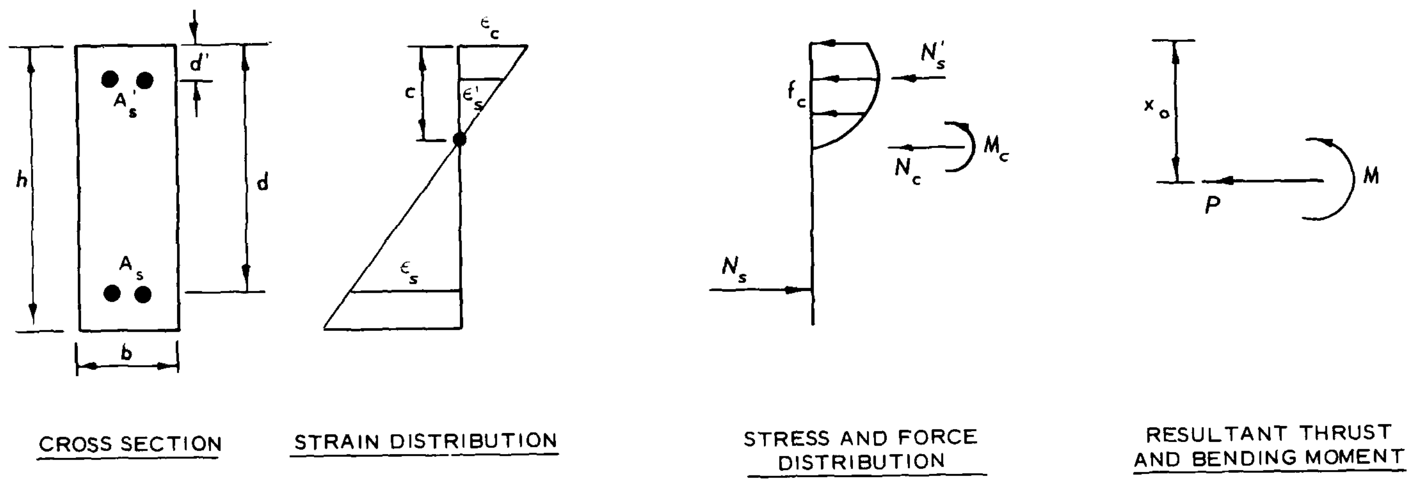

Figure 2.1 Determination of thrust and bending moment resisted by reinforced concrete cross section.

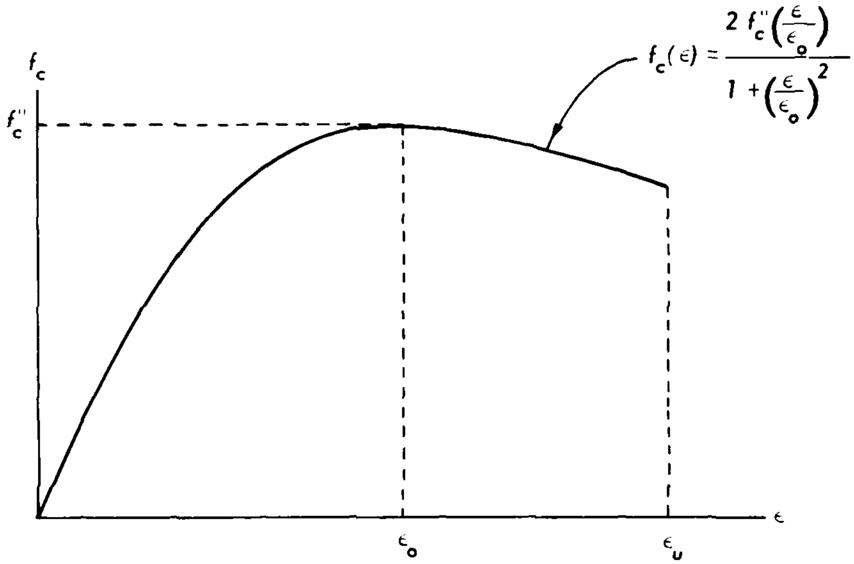

Figure 2.2 Assumed stress-strain relation for concrete.

SECTION 3

IMPLICIT LINEAR STATISTICAL ANALYSIS

Through all the preceding Equations 2.1-2.19, the resistance of a reinforced concrete beam-column has been expressed as an implicit function of a set of parameters describing the concrete strength, section geometry, reinforcement strength, and its placement. Since these parameters are random variables in practice, it follows that the resistance is a random variable. Now, if a random variable Y is an explicit function of a set of n random variables X_i ,

$$Y = Y(X_1, X_2, \ldots X_n) \tag{3.1}$$

then approximations have been derived for the moments of Y in terms of functions of the moments of the independent variables X_i .[5] For example, if the multidimensional Taylor-series expansion of $Y(X_1, X_2, \ldots X_n)$ about the means $(\mu_1, \mu_2, \ldots \mu_n)$ of the constituent variables is truncated after the linear terms, it can be shown that the expectation of Y is approximately

$$E\{Y\} \approx Y(\mu_1, \mu_2, \ldots \mu_n) \tag{3.2}$$

and that its variance is approximately

$$\mathrm{Var}\{Y\} \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{\partial Y}{\partial X_i} \cdot \frac{\partial Y}{\partial X_j}\right)_{(\mu_1, \mu_2, \ldots \mu_n)} \mathrm{Cov}\{X_i, X_j\} \tag{3.3}$$

In Equation 3.2, the mean of a function of random variables is approximated to first order by the functional relation of these variables evaluated at their mean values. Equation 3.3 indicates that each pair of the constituent random variables (X_i, X_j) contributes to the dispersion of Y in a manner proportional to their own covariance $\mathrm{Cov}\{X_i, X_j\}$

and proportional to a factor $\left(\frac{\partial Y}{\partial X_i} \cdot \frac{\partial Y}{\partial X_j}\right)_{(\mu_1, \mu_2, \ldots \mu_n)}$, which is a first order measure of the sensitivity of changes in Y to those in X_i and X_j .

However, this approach has heretofore not been applied to the resistance of a reinforced concrete beam-column since an explicit expression for this resistance does not generally exist from which to evaluate the partial derivatives of resistance with respect to the constituent variables $\frac{\partial Y}{\partial X_i}$. It is here proposed to evaluate these derivatives from the following finite difference expression

$$\frac{\partial Y}{\partial X_i} \approx \frac{Y(\mu_1, \ldots \mu_i + k\sigma_i, \ldots \mu_n) - Y(\mu_1, \ldots \mu_i - k\sigma_i, \ldots \mu_n)}{2k\sigma_i} \qquad i = 1, \ldots n \qquad (3.4)$$

which can be obtained for an implicit functional relation as readily as for an explicit one. In some cases, it may be more convenient to employ Equation 3.4 even when an explicit relation exists from which to obtain $\frac{\partial Y}{\partial X_i}$. Since the ultimate goal of this analysis is not necessarily to precisely evaluate $\frac{\partial Y}{\partial X_i}$ but rather to reasonably approximate the functional relation between Y and X_i throughout the range of the random variables X_i , the finite difference Equation 3.4 has herein been calculated from the functional values at $k = 1$, which are one standard deviation σ_i above and below the mean values of X_i . Note that through the individual terms of Equation 3.3 this implicit linear statistical analysis readily permits one to evaluate the relative contributions of the individual constituent random variables X_i to the variation of the dependent variable Y . As is the case with the explicit linear statistical analysis in Equations 3.2 and 3.3, the accuracy of the implicit linear statistical analysis in Equations 3.2 through 3.4 is determined by the dispersion of the X_i and by the severity of any nonlinearity in the functional relation.

To illustrate, consider the univariate function $Y = X^2$ shown in Figure 3.1. If X is uniformly distributed on the interval (4,6), it readily follows that for this explicit function

$$E\{Y\} = \int_4^6 x^2(1/2)\,dx = 25.33$$

and the coefficient of the variation is

$$V\{Y\} = \frac{\sqrt{\int_4^6 x^4(1/2)\,dx - E^2\{Y\}}}{E[Y]} = 0.2282$$

For this case, the implicit linear statistical analysis, Equations 3.2 through 3.4 imply that $E\{Y\} \approx 25.00$ and $V\{Y\} \approx 0.2309$. If, however, X is uniformly distributed on the interval (0,2), $E\{Y\} = 4/3 = 1.333$ and $V\{Y\} = 2/\sqrt{5} = 0.8944$, while Equations 3.2 through 3.4 yield $E\{Y\} \approx 1.000$ and $V\{Y\} \approx 1.1547$. Figure 3.1 indicates that in both cases the implicit linear analysis approximates the functional relation $Y(X)$ by a straight line passing through the point $(\mu_X, Y(\mu_X))$ and having the same slope as the straight line passing through the points $(\mu_X + \sigma_X, Y(\mu_X + \sigma_X))$ and $(\mu_X - \sigma_X, Y(\mu_X - \sigma_X))$. In the former case, this is a reasonable approximation to $Y(X)$ in the range of the random variable X, while in the latter case the nonlinearity of $Y(X)$ is more severe in the range of X.

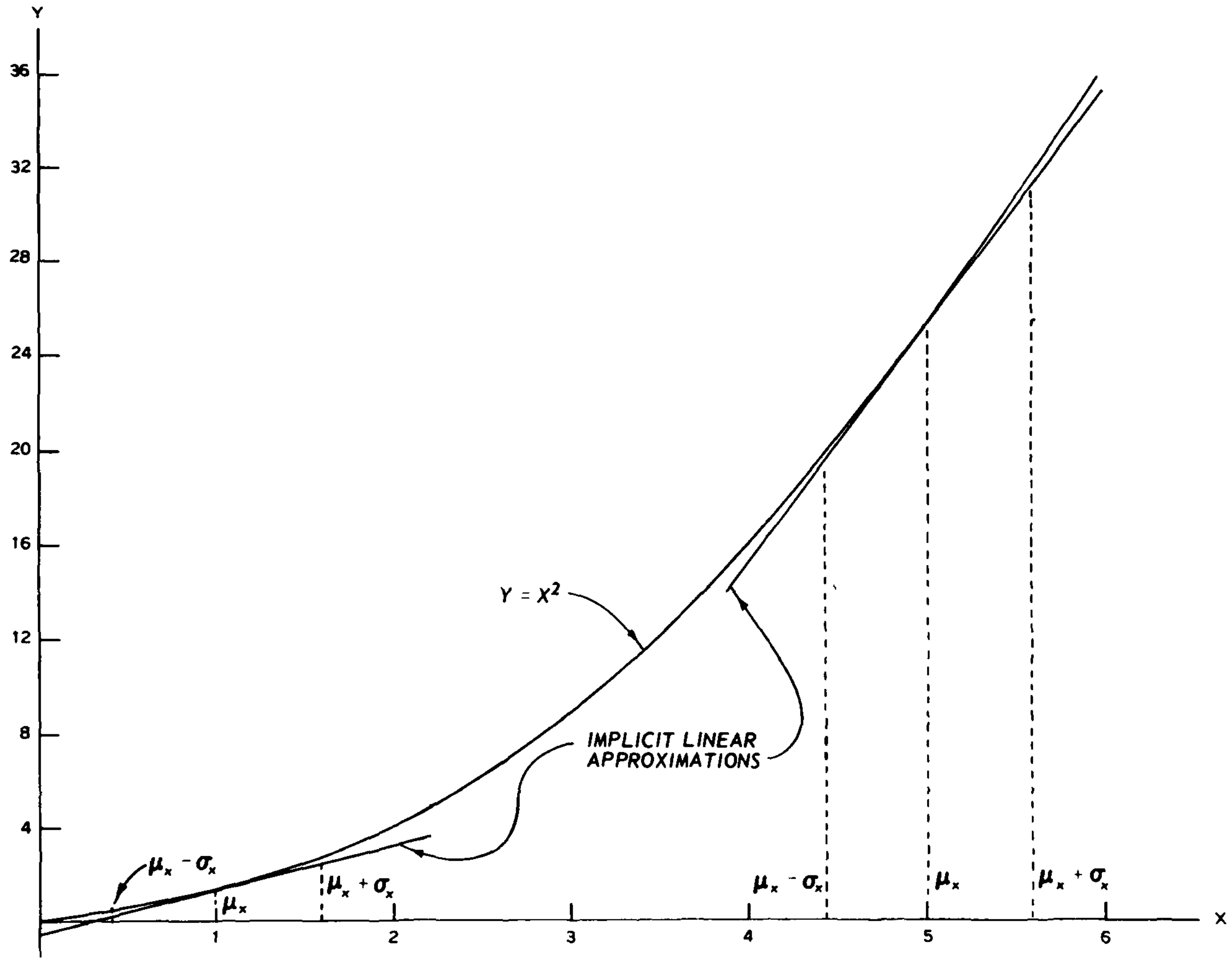

Figure 3.1 $Y = X^2$ and implicit linear approximations.

SECTION 4

VARIABILITY OF RESISTANCE DUE TO BASIC FABRICATION RANDOMNESS

Since Equations 1 through 19 implicitly relate the resistance of a reinforced concrete beam-column to 11 random variables describing the concrete strength, section geometry, reinforcement strength, and its placement, the implicit linear analysis of the previous section can be used to determine the uncertainty in this resistance. The first cross section to be analyzed is described in the first three columns of Table 4.1. The constituent variables X_i that describe this cross section are assumed to be uncorrelated, and the given coefficients of variation are representative of those encounted for average quality in fabrication and construction.[2,6]

The mean moment-thrust interaction curve obtained from this analysis appears in Figure 4.1. Also shown in this figure is the corresponding mean curve obtained from a previously published Monte Carlo simulation in which a random number generator repeatedly selects values of the X_i from which a series of interaction curves are determined.[2] The small biased difference between the two results is attributed to the failure criteria and the concrete stress distribution assumed herein, which differ slightly from those used in Reference 2.

In Figure 4.2, $V\{R\}$ is presented as a function of

$$\alpha \equiv \tan^{-1} h/e \tag{4.1}$$

which is a measure of the eccentricity of the loading. Once again, the result of the implicit linear analysis and that of the previously published Monte Carlo simulation are in agreement. This plot of the coefficient of variation of resistance due to basic fabrication randomness plays an important role in the determination of any reliability based design safety factor.[7,8,9] Of further interest are the individual contributions to the variability of resistance made by the constituent variables X_i at different eccentricities, which are presented in

the last six columns of Table 4.1. For each variable X_i , this table gives the value of $\frac{\partial R}{\partial X_i} \cdot \frac{\mu_i}{E\{R\}}$, which is a dimensionless measure of the sensitivity of the resistance to the particular variable, and the value of $\left|\frac{\partial R}{\partial X_i}\right| \cdot \frac{\sigma_i}{E\{R\}}$, which would be the coefficient of variation of resistance if all the other variables were deterministic. These values are presented for the pure moment loading $\alpha = 0$, for the balanced loading $\alpha = 1.033$ in this case, and for the pure thrust loading $\alpha = \pi/2$. From this tabulation, it is seen that the variability in concrete strength f''_c contributes most of the variability in resistance for the pure thrust loading, but that this contribution diminishes with increasing eccentricity. On the other hand, most of the variability in resistance for the pure moment loading is due to the variability in the placement of the tensile reinforcement d and in the reinforcement strength f_y . It is further apparent that these contributions diminish with decreasing eccentricity. For the balanced loading, all three of these variables contribute significantly to the uncertainty in resistance.

Table 4.1. Individual Contributions to Variability of Resistance Due to Basic Fabrication Randomness

X_i	μ_i	σ_i/μ_i	$\alpha = 0$: $\frac{\partial R}{\partial X_i} \cdot \frac{\mu_i}{E\{R\}}$	$\alpha = 0$: $\left\lvert\frac{\partial R}{\partial X_i}\right\rvert \frac{\sigma_i}{E\{R\}}$	$\alpha = 1.033$: $\frac{\partial R}{\partial X_i} \cdot \frac{\mu_i}{E\{R\}}$	$\alpha = 1.033$: $\left\lvert\frac{\partial R}{\partial X_i}\right\rvert \frac{\sigma_i}{E\{R\}}$	$\alpha = \pi/2$: $\frac{\partial R}{\partial X_i} \cdot \frac{\mu_i}{E\{R\}}$	$\alpha = \pi/2$: $\left\lvert\frac{\partial R}{\partial X_i}\right\rvert \frac{\sigma_i}{E\{R\}}$
f''_c	2.975 ksi	0.15	2.208E-002	3.312E-003	5.356E-001	8.034E-002	6.476E-001	9.713E-002
ε_o	0.002	0.13	-1.041E-002	1.353E-003	7.274E-002	9.456E-003	2.340E-001	3.042E-002
ε_u	0.004	0.16	2.600E-004	4.160E-005	1.849E-002	2.958E-003	-2.029E-001	3.246E-002
b	20 in	0.04	2.397E-002	9.586E-004	5.812E-001	2.325E-002	6.560E-001	2.624E-002
h	20 in	0.04	-1.001E+000	4.006E-002	-3.141E-001	1.256E-002	6.348E-001	2.539E-002
f_y	47.7 ksi	0.09	9.873E-001	8.886E-002	4.002E-001	3.602E-002	3.366E-001	3.030E-002
E_s	29,000 ksi	0.03	-1.047E-002	3.141E-004	8.355E-002	2.506E-003	2.439E-002	7.318E-004
A_s	6.00 in^2	0.03	9.528E-001	2.859E-002	1.564E-001	4.691E-003	1.923E-001	5.768E-003
A'_s	6.00 in^2	0.03	2.318E-002	6.955E-004	2.829E-001	8.488E-003	1.515E-001	4.544E-003
d	18 in	0.07	1.085E+000	7.594E-002	9.374E-001	6.562E-002	3.083E-002	2.158E-003
d'	2 in	0.07	-5.843E-002	4.090E-003	-2.636E-002	1.845E-003	3.272E-003	2.291E-004
			E{R} = 225.9 kips		E{R} = 692.7 kips		E{R} = 1614 kips	
			V{R} = 0.1269		V{R} = 0.1138		V{R} = 0.1172	

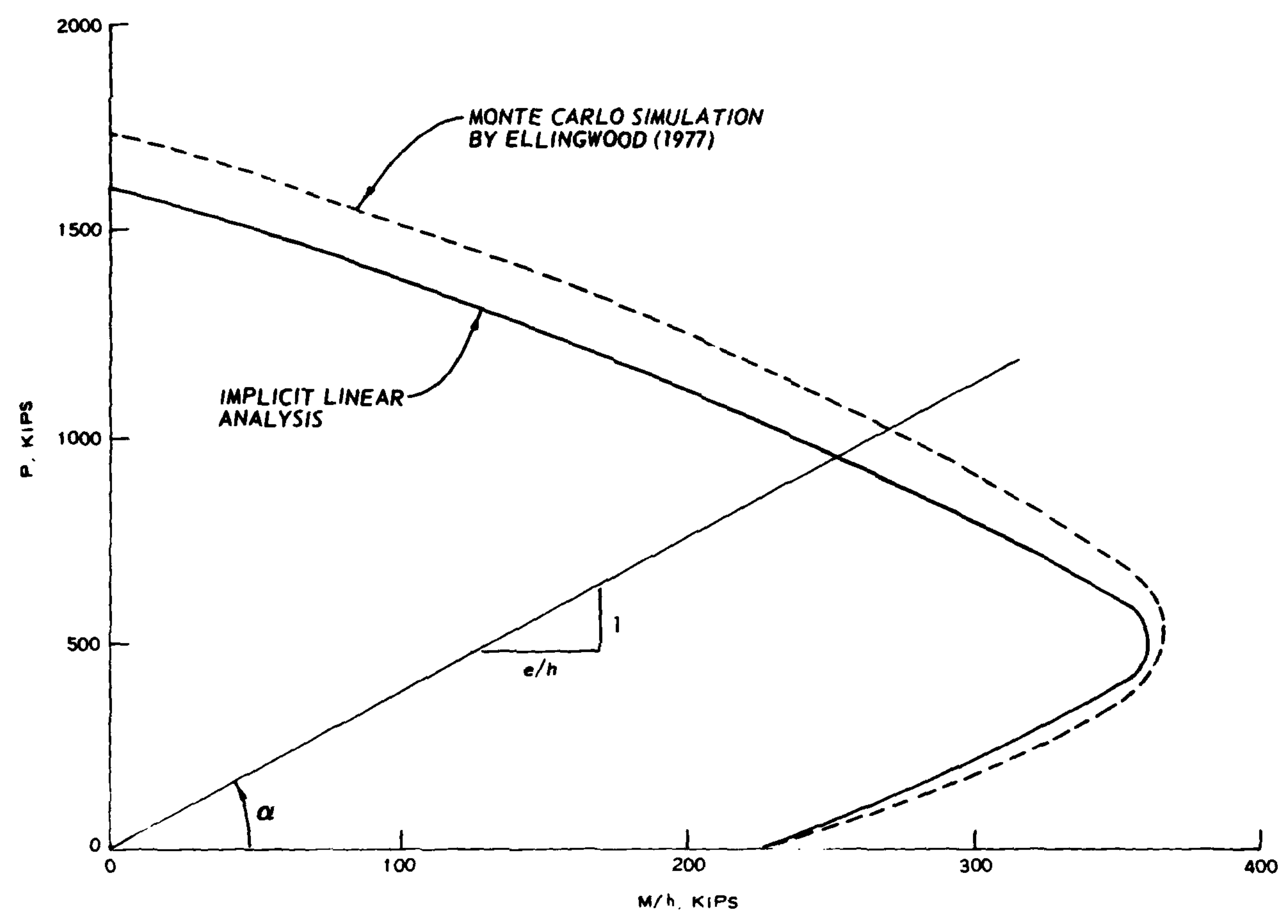

Figure 4.1 Mean beam-column resistance considering basic fabrication randomness.

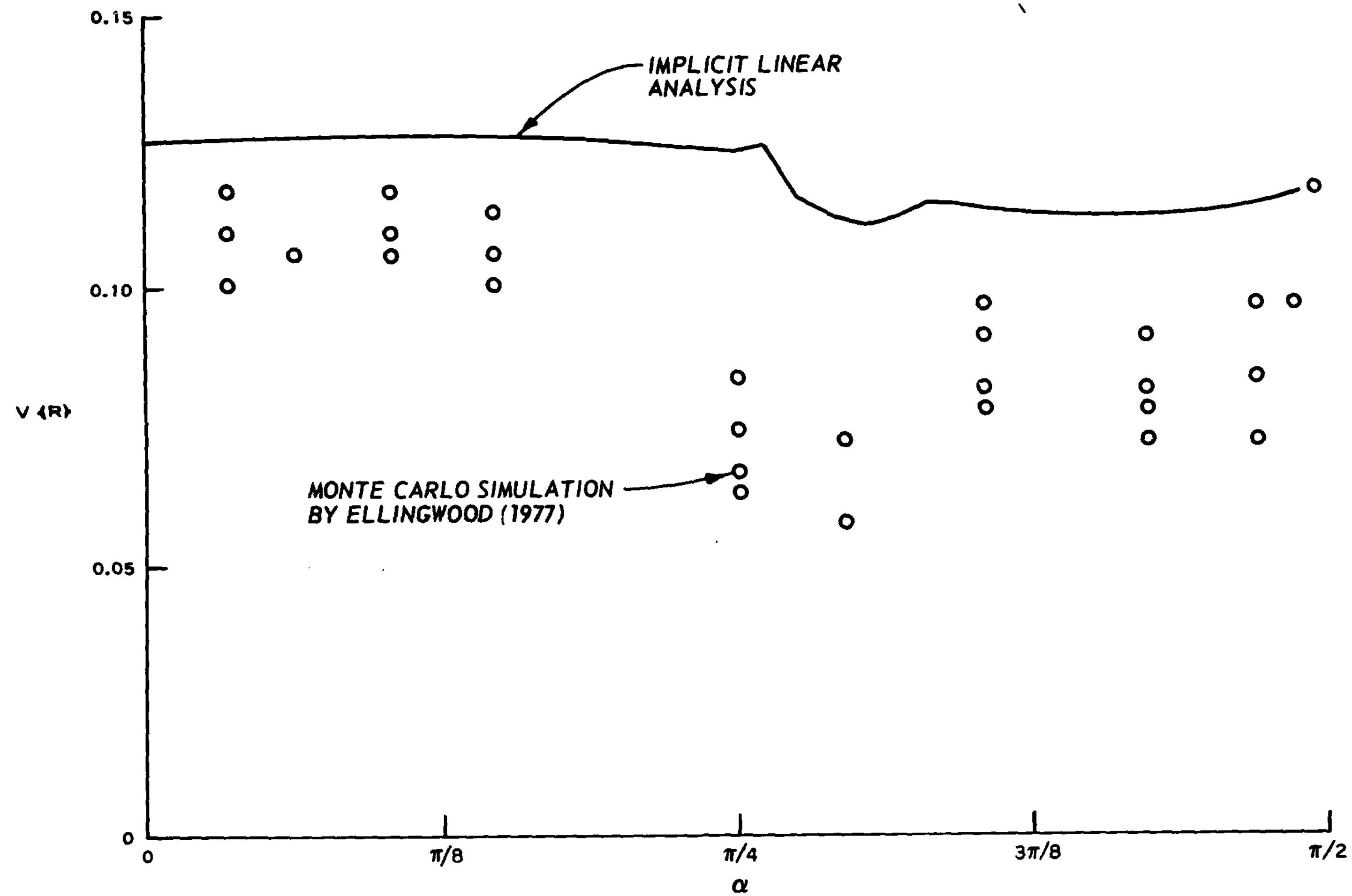

Figure 4.2 Variation in beam-column resistance due to basic fabrication randomness.

SECTION 5

VARIABILITY OF RESISTANCE DUE TO ESTIMATION UNCERTAINTY

In the previous section, the variability of beam-column resistance R due to the basic randomness associated with typical fabrication practices was modeled. In some cases, the mean values of the individual parameters describing a reinforced concrete cross section must themselves be estimated, thus introducing further uncertainty about the resistance. An example of this is the estimation of the resistance of a structural target that must be made from relatively incomplete information. Another instance occurs in the condition survey of an existing structure of uncertain design. Such uncertainties will now be discussed using the implicit linear analysis developed earlier.

The cross section chosen for this study is described in the first three columns of Table 5.1 and represents a 1-inch-wide section from the roof of a hardened reinforced concrete rectangular box structure described in Reference 10. The mean values μ_i of the constituent variables X_i represent a hypothetical best estimate of the values of these variables in the prototype structure. The uncertainty associated with the estimation of f''_c, f_y, A_s, and A'_s from only limited data about this particular structural design is illustrated by assigning a coefficient of variation of 0.5 to these variables. However, reinforced concrete design practice in general indicates that the coefficients of variation of the remaining variables are simply those associated with basic fabrication randomness, which were given in Table 4.1. For example, whatever the particular but unknown design, the tensile reinforcement will be placed as deep as possible; thus the randomness in d is due only to placement errors encountered in construction practice. As in the case of basic fabrication randomness, the X_i are assumed to be uncorrelated for this cross section.

The mean moment-thrust interaction curve obtained for this structure from the implicit linear analysis is shown in Figure 5.1. To verify this analysis, three different Monte Carlo simulations of 100

replications each were conducted. In these simulations, the X_i were assumed to be independent and normally distributed with the means and coefficients of variation given in Table 5.1. Each distribution was truncated to eliminate negative values, and the constraints $d < h$ and $\varepsilon_o < \varepsilon_u$ were also imposed. The results of these Monte Carlo simulations are plotted in Figure 5.1 for comparison with those of the implicit linear analysis. It is noted that the agreement between the two results is not as good near the balanced loading as it is elsewhere. This is attributed to some nonlinear behavior of the resistance function near this balanced loading. In Figure 5.2, the difference between the implicit functional dependence of R on f_y at this loading and the linear approximation employed herein can be seen. Smaller differences between the actual and the approximate behaviors also exist for the variables f''_c and A_s at this loading.

In Figure 5.3, the coefficient of variation of resistance $V\{R\}$ is displayed as a function of the eccentricity of the loading. Some agreement between the linear analysis and the Monte Carlo simulations is obtained notwithstanding the nonlinearity noted in the previous paragraph. For the assumptions made about the uncertainties of this cross section, the variability of resistance is seen to generally decrease with decreasing eccentricity. This is further understood by examining the individual contributions to this variability made by the constituent variables X_i in Table 5.1. As noted, the uncertainty about concrete strength f''_c contributes most of the uncertainty about resistance at small eccentricities, but this contribution diminishes with increasing eccentricity. However, most of the variability in resistance at large eccentricities is due to the uncertainty in the estimation of f_y and A_s, and these contributions diminish with decreasing eccentricity. At the balanced loading, the contributions of all three of these variables to the uncertainty of resistance are significant.

Table 5.1. Individual Contributions to Variability of Resistance Due to Uncertainty in Estimation of Individual Parameters

			$\alpha = 0$		$\alpha = 1.099$		$\alpha = \pi/2$	
X_i	μ_i	σ_i/μ_i	$\frac{\partial R}{\partial X_i} \cdot \frac{\mu_i}{E\{R\}}$	$\left\lvert\frac{\partial R}{\partial X_i}\right\rvert \frac{\sigma_i}{E\{R\}}$	$\frac{\partial R}{\partial X_i} \cdot \frac{\mu_i}{E\{R\}}$	$\left\lvert\frac{\partial R}{\partial X_i}\right\rvert \frac{\sigma_i}{E\{R\}}$	$\frac{\partial R}{\partial X_i} \cdot \frac{\mu_i}{E\{R\}}$	$\left\lvert\frac{\partial R}{\partial X_i}\right\rvert \frac{\sigma_i}{E\{R\}}$
f''_c	4.930 ksi	0.50	5.815E-002	2.908E-002	5.396E-001	2.698E-001	7.627E-001	3.813E-001
ϵ_o	0.002	0.13	-3.295E-002	4.284E-003	8.470E-002	1.101E-002	2.686E-001	3.491E-002
ϵ_u	0.004	0.16	-8.852E-005	1.416E-005	4.887E-002	7.819E-003	-2.258E-001	3.612E-002
b	1.000 in	0.04	5.754E-002	2.302E-003	6.448E-001	2.579E-002	7.795E-001	3.118E-002
h	20.72 in	0.04	-1.081E+000	4.005E-002	-2.008E-001	8.032E-003	7.593E-001	3.037E-002
f_y	71.60 ksi	0.50	3.762E-001	4.881E-001	3.845E-001	1.922E-001	1.771E-001	8.856E-002
E_s	29,000 ksi	0.03	-3.236E-002	9.707E-004	1.172E-001	3.515E-003	3.746E-002	1.124E-003
A_s	0.2072 in^2	0.50	9.455E-001	4.732E-001	4.069E-001	2.035E-001	1.098E-001	5.488E-002
A'_s	0.2072 in^2	0.50	-7.841E-004	3.921E-004	1.549E-001	7.747E-002	1.117E-001	5.583E-002
d	17.76 in	0.07	1.083E+000	7.581E-002	5.016E-001	6.311E-002	1.933E-002	1.353E-003
d'	2.96 in	0.07	-3.058E-002	2.141E-003	-3.672E-002	2.570E-003	1.739E-003	1.217E-004
			E{R} = 11.25 kips		E{R} = 47.85 kips		E{R} = 118.7 kips	
			V{R} = 0.6859		V{R} = 0.4026		V{R} = 0.4047	

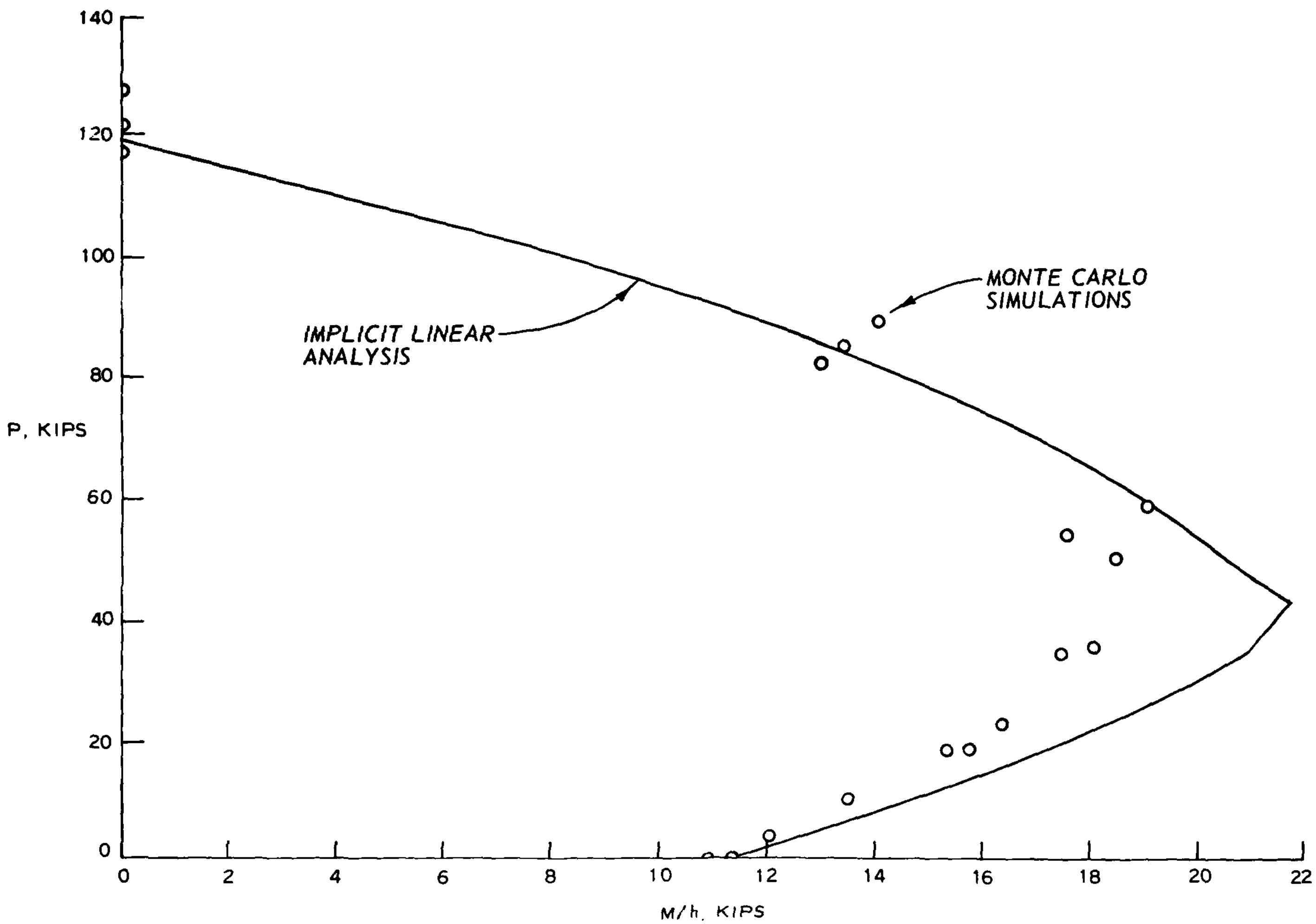

Figure 5.1 Mean beam-column resistance considering uncertainty in e-timation of individual parameters.

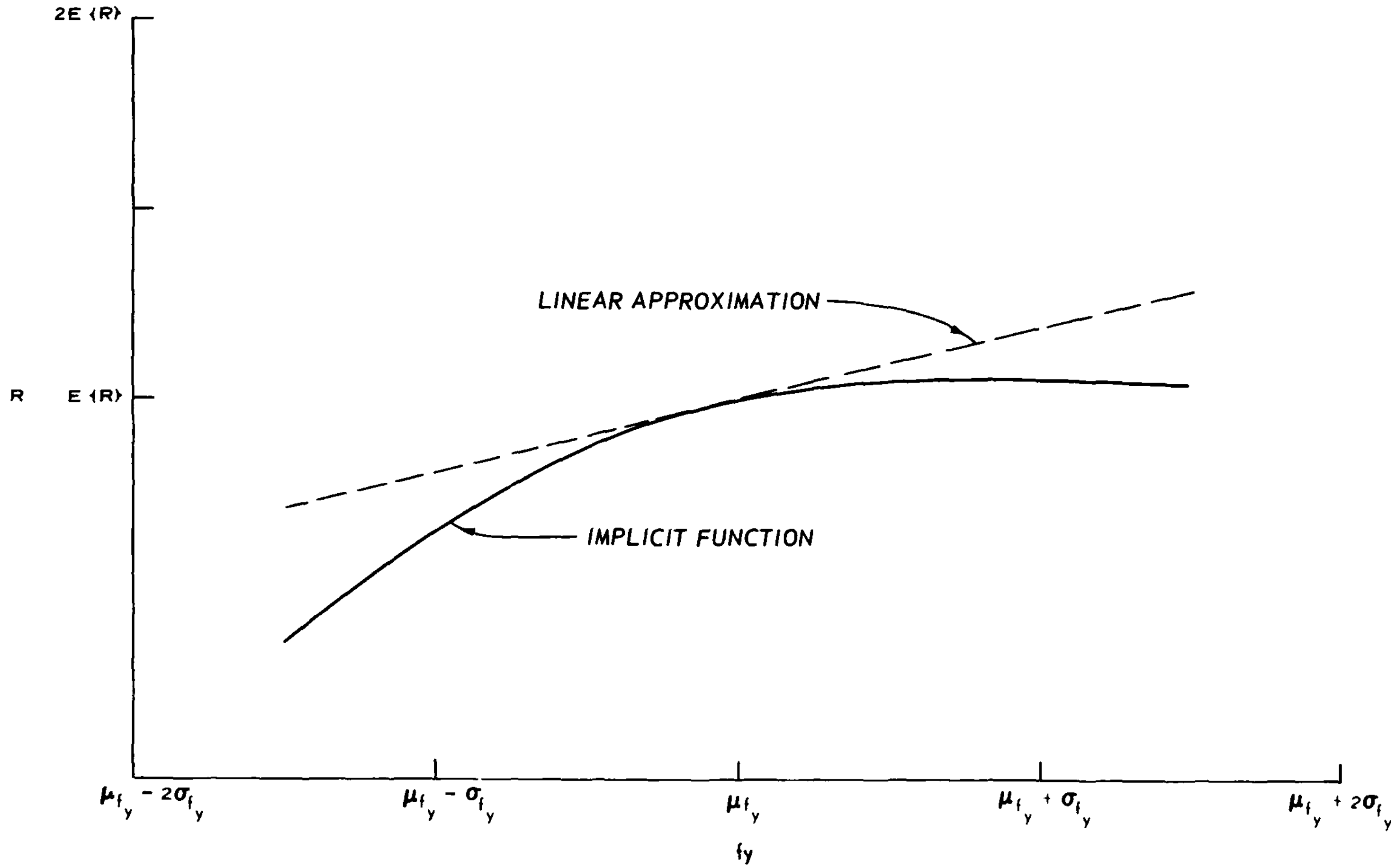

Figure 5.2 Sensitivity of resistance for balanced loading to uncertainty in estimation of f_y .

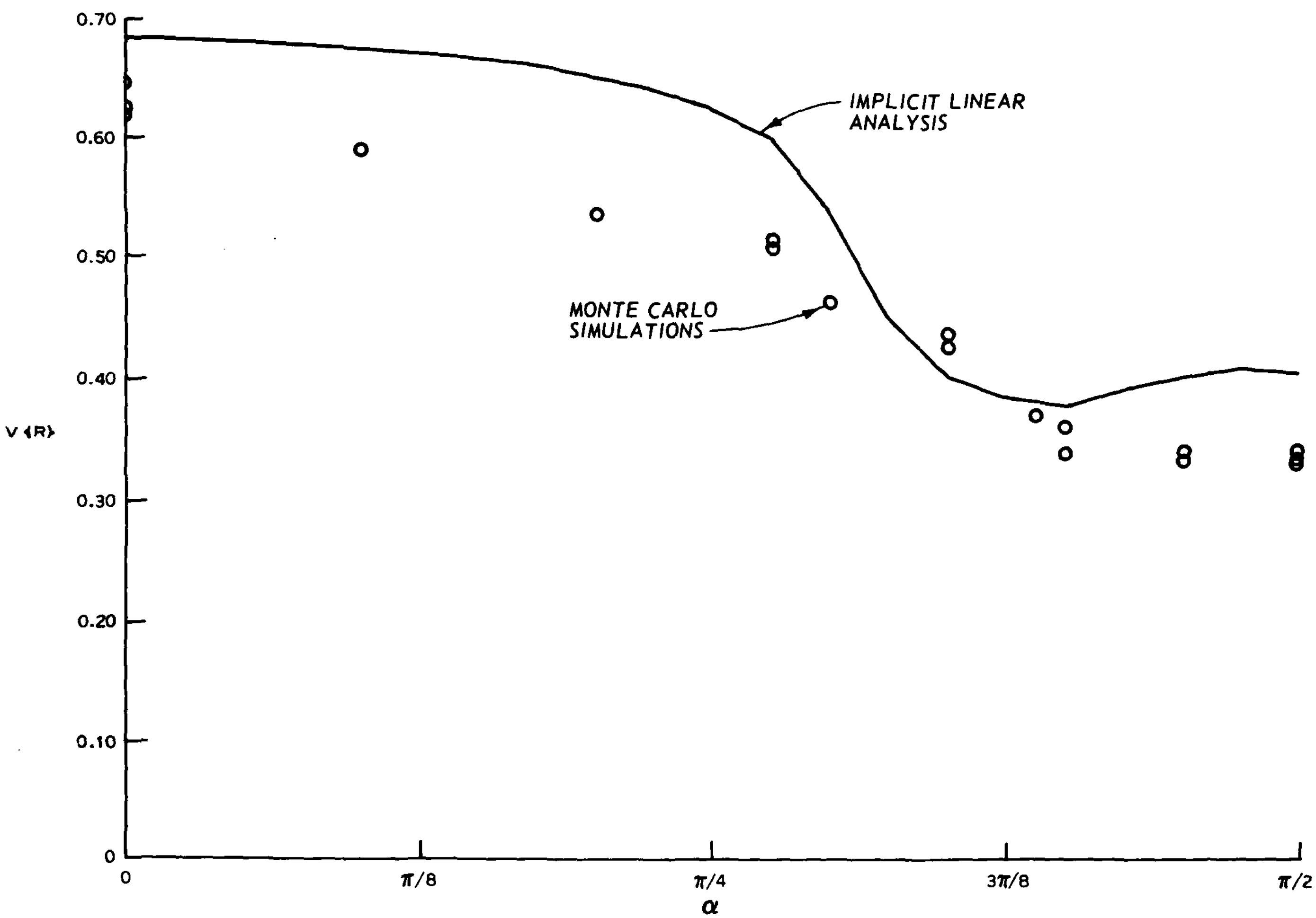

Figure 5.3 Variation in beam-column resistance due to uncertainty in estimation of individual parameters.

SECTION 6

ADDITIONAL VARIABILITY OF RESISTANCE DUE TO IMPERFECT PREDICTION EQUATION

In the preceding sections, the variability of beam-column resistance due to basic fabrication randomness and to estimation uncertainty has been examined. There exists a further source of uncertainty in resistance due to various factors omitted from the deterministic analysis. To quantify this uncertainty, the actual resistance is assumed to be

$$R_a = \beta R + \delta \tag{6.1}$$

in which R is the resistance computed from Equation 2.19, β is a dimensionless factor representing the bias of the deterministic analysis, and δ is a zero-mean random variable representing the variability of resistance due to factors neglected in this analysis that are inherently random. From Equation 6.1, it follows that

$$E\{R_a\} = \beta \cdot E\{R\} \tag{6.2}$$

and if δ is assumed to be independent of the 11 constituent random variables determining R

$$V^2\{R_a\} = V^2\{R\} + \frac{\mathrm{Var}\{\delta\}}{\beta^2 E^2\{R\}} \tag{6.3}$$

From experiments on eccentrically loaded rectangular beam-columns[11] in which the constituent variables determining R were measured, it is estimated that $\beta = 0.97$ and $\frac{\mathrm{Var}^{1/2}\{\delta\}}{\beta E\{R\}} = 0.061$.[2] Thus, the coefficient of variation of the actual resistance of the beam-column of Table 4.1 at the balanced loading is $V\{R_a\} = \sqrt{(0.1138)^2 + (0.061)^2} = 0.1291$. This indicates that various factors omitted in the

deterministic analysis of Equations 2.1 through 2.19 contribute some small additional uncertainty in beam-column resistance beyond that associated with basic fabrication randomness. On the other hand, the contribution of these factors to the variability of resistance beyond that due to estimation uncertainty is less important. For example, at the balanced loading of the beam-column of Table 5.1, $V\{R_a\} = \sqrt{(0.4026)^2 + (0.061)^2} = 0.4072$.

SECTION 7

CONCLUSIONS

An approximate linear statistical analysis has been developed for the moments of an implicit function of jointly distributed random variables. The application of this analysis to a reinforced concrete beam-column indicates that the variability of resistance caused by basic fabrication randomness is due largely to variability in concrete strength at small eccentricities but is due increasingly to variability in reinforcement strength and in the placement of tensile reinforcement at larger eccentricities. The uncertainty about the resistance is larger if the mean parameters of the cross section must themselves be estimated. For large eccentricities, this uncertainty is governed by the uncertainties about reinforcement strength and the amount of tensile reinforcement, but for small eccentricities, the uncertainty is determined by the uncertainty about concrete strength. The available experimental data indicate that the additional uncertainty in resistance due to various factors omitted from these analyses is of some small significance in the case of basic fabrication randomness and of even less importance in the case of estimation uncertainty.

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APPENDIX A

NOTATION

A_s	Area of tensile reinforcement
A'_s	Area of compressive reinforcement
b	Width of cross section
c	Distance from extreme compressive fiber to neutral axis
c_b	Distance from extreme compressive fiber to neutral axis for balanced loading
$Cov\{\ \}$	Covariance of two random variables
d	Distance from extreme compressive fiber to centroid of tensile reinforcement
d'	Distance from extreme compressive fiber to centroid of compressive reinforcement
e	Eccentricity of load = M/P
E_s	Modulus of elasticity of reinforcement
$E\{\ \}$	Expectation of a random variable
f_c	Concrete stress
f''_c	Compressive strength of concrete in reinforced concrete member
f_s	Reinforcement stress
f_y	Yield strength of reinforcement
h	Depth of cross section
i, j	Index of functionally independent random variables
k	Number of standard deviations above and below mean at which function is evaluated in implicit linear analysis
M	Resultant bending moment about plastic centroid
M_c	Bending moment of concrete stress distribution about neutral axis
n	Number of constituent random variables
N_c	Resultant concrete force
N_s	Resultant force in tensile reinforcement
N'_s	Resultant force in compressive reinforcement
N_{co}	Resultant concrete force for thrust without bending moment
N_{so}	Resultant force in tensile reinforcement for thrust without bending moment

N'_{so}	Resultant force in compressive reinforcement for thrust without bending moment
P	Resultant axial thrust
R	Beam-column resistance defined by Equation 2.19
R_a	Actual beam-column resistance
$V\{\ \}$	Coefficient of variation of a random variable
$Var\{\ \}$	Variance of a random variable
X_i	Functionally independent random variables
x_o	Distance from extreme fiber to plastic centroid
Y	Functionally dependent random variable
α	Angular measure of eccentricity = $\tan^{-1} h/e$
β	Bias between actual and computed resistance
δ	Random variable representing difference between computed unbiased resistance and actual resistance
ϵ	Strain
ϵ_c	Maximum compressive concrete strain
ϵ_m	Minimum compressive concrete strain
ϵ_o	Concrete strain at the maximum stress f''_c
ϵ_s	Tensile reinforcement strain
ϵ'_s	Compressive reinforcement strain
ϵ_u	Useful limit of compressive strain in concrete
μ_i	Mean of X_i
σ_i	Standard deviation of X_i